: ARIST. DEMAGNITX

```
PROPOSITIO. VIII.
```

Cumfol totus deficit; tunc idem conus somprebendit folem \& lunam, ad vifum no frum ruerticem babens.

Quoniam enim fi deficiat fol, ob lunx oppofitio nem deficit.incidit autem fol in conum lunam com prehendenrem, qui ad vifum noftrum verticem habet.vel igitur fol ipfi cono congruit, vel excedit,vel ab eo exceditur, \& fi quidem excedit, non deficiet totus, fed eminebit ipfius pars excedens, fi vero ab eo exceditur, permanebit folis defectus, quoad partem illam, qua exceditur, pertranfiuerif. atqui deficit totus, $\&$ non permanet deficiens. illud enim ex obferuatione manifeftŭ eft. quare neq; excedit, neq;exceditur.ipfi igitur cögruat neceffe eft. \& cöprehendetur à cono lunam comprehendente, gui ad vifum noftrum verticem habet.

## PROPOSITIO. IX

Solis diameter maior eft, quàm duodeui gintupla diametri lune: minor vero quàm rigintupla.

Sit nofter quidem vifus ad $A$; folis autem cenerum B, \&olune centrnm C.tquando conus folem e lunam comprehédés ad vifum noftrum verticern habeat, hocef quando punctacB fint in eadera recta linea \& per ACB.planam producatur, qued faciet fectiones in fpheqris quidem maximos circu-

## PROPOSITION VIII

When the sun is totally eclipsed, then one and the same cone, which has its vertex at our eye, comprehend the sun and the moon.

If the sunlight lacks, it lacks because the moon is in opposition; at that time the sun falls into the cone comprehending the moon and having its vertex at our eye. In fact or the sun coincides exactly with the same cone, or it surmounts it, or it falls short of it. If also the sun surmounts it, itself would not be totally eclipsed, but the exceeding portion of himself stands out; however if it falls short, the sunlight will be missing for the time which it takes to pass through the portion by which it falls short, and although it is totally eclipsed it not does remain missing, this is manifest from observation, therefore it can neither surmount nor fall short; consequently it must exactly coincide with the cone, and will be comprehended by the cone comprehending the moon and having its vertex at our eye

## PROPOSITION IX

The diameter of the sun is greather than 18 times, but less than 20 times, the diameter of the moon.

Let our eye be at $A$, while let $B$ the centre of the sun, and $C$ the centre of the moon when the cone comprehending both the sun and the moon has its vertex at our eye, that is, when te points $\mathrm{A}, \mathrm{C}, \mathrm{B}$ are in a straight line. Let a plane be carried through ACB which will cut the spheres in great circles

## ETDIST. SOL, ETLVNAE. ES

los, in cono autem reCtas lineas. faciat igitur in fphxris ma ximos circulos FG, KLH : \& in cono reCtas lineas AFH, A GK, \& CG, BKiun gantur. erit ut BA ad AC, ita BK ad C G. fed BA ipfius A C oftenfa eft maior, quidē,quã duodeut gitupla,minorvero, quâ uigintupla .ergo \& BK maior erit, quā duodeuigintu pla ipfius CG, \& mi nor,quă vigintupla. PROPOSITIO.
$x$.
Sol ad lunam
suaiorem proportionē babet, quat
 5832 ad $I$, minore vero quàm 8000 ad $r$ :

Sit folis quidē diameter A ; lune vero diameter B . $3 \boldsymbol{3}$ node ergo $A$ ad $B$ maiorē proportionē hēt, quā 18 ad 1 , comide $\&$ minoré quā 20 ad 1 . Et qñ cubus, qui fit ex $A$ ad cubum qui ex $B$ triplà proportioné hér eius, quā $\mathbf{A}$ 28. duo häbet ad B:habet autcm \& Sphara circa diametrum E. Anad
and the cone in streight lines. Then let it generate on the spheres the great circles FG\& KLH and on the cone the straight lines AFH\& AGK, let C\&G and B\&K be joined. As BA is to $A C$, so will $B K$ be to CG, but it was proved that BA is greater than 18 times, but less than 20 times,AC. Therefore BK will also greater than 18 times, but less than 20 times, CG.


A

The sun has to the moon a ratio greater than that which 5832 has to 1, but less than that which 8000 has to 1 .

Let A be the diameter of the sun, B that of the moon; then A has to B a ratio greater than that which 18 has to 1 , but less than that which 20 has to 1 . As the cube on A has to the cube on B the ratio triplicate of that which A has to B,

- ARIT. DEMAGN.
A ad fpharam circa diametrum $B$ triplam propottionem eius, quam habet $A$ ad $B$. eft igitur $v t \mathrm{ch}^{-}$


## A

B
 A ad fphxram circa diametrum B: fed/cubus ex $\boldsymbol{A}$ ad cubum ex $B$ maiorem proportionem habet, $q \boldsymbol{q} \overline{2}$ 5832 ad 1 , minorem vero quàm 8000 ad r , quioniz $A$ ad $B$ maiorem proportionem haber, quàm 18 ad 1; \& minorem, quàm 20 ad i.ergo \& fol ad lunam maiorem proportionem habebit, quā 5832 ad $\mathrm{r}, \mathrm{mi}$ norem vero, quàm 8000 ad I .

> PROPOSITIONXI
Luno diameter, minor est, quam dua quadragefime quinte partes, maior vero, quàm pars trigefima diftantig, qua contrum lune à vifu noftro diftat.
Sit enim nofter vifus ad $A, \&$ lunx cêtrum $B$, quä do convs folem, \&: lunam comprehendens ad vifum noflrum verticem habeat. Dico fieri ea.qux in propoftione continentur, iungatur enim AB, \& per iptam planump producatur, quod faciet in iphara circulum, in cono autem rectas lineas faciat igirur in fphęra circulum CED: $\&$ in cono rectas lineas. AD, AC:iungaturóne CB \& ad E producaturi. itaque có ftat ex ea,quod demontratum eft, angulum BAC dimidij
then the sphere about the diameter A also will have to the sphere about the diameter B, the ratio triplicate of that which A has to B .

A

B I

Therefore, as the cube on A is to the cube on B , so the sphere about tha diameter A is to the sphere about the diameter B ; but the cube on A has to the cube on B a ratio greater than that which 5832 has to 1, but less than that which 8000 has to 1 , since A has to B a ratio greater than that which 18 has to 1 , but less than that which 20 has to 1 ; consequently the sun will have to the moon a ratio greater than that which 5832 has to 1 , but less than that which 8000 has to 1 .

## PROPOSITION XI

The diameter of the moon is less than two forty-five parts, but greater than thirtieth part of the distance of the centre of the moon from our eye.

Let our eye be at A , and let B the centre of the moon when the cone comprehending booth the sun and the moon has its vertex at our eye. I say that all that is written in the above proposition takes place. Indeed let $A \& B$ be joined and let the plane through $A B$ be drawn, which will cut the sphere in a circle and the cone in the straight lines, then this plane cut the sphere in the circle CED and the cone in the staight lines $\mathrm{AD}, \mathrm{AC}$; let $\mathrm{C} \& \mathrm{~B}$ joined and produced to E . Then it is manifest from what has before been proved that the angle BAC is the forty-fifth part of half a right angle ${ }^{\text {A }}$, and

## ETDIST:SOYETLVNAE 19

dimidj recti effe parten quadragefimam quintâ:
 ; quadrayjefinaín quintam partemanulo igitur mi: moneft SC, quàp quadragefima quintapansiphis


B A.eftǵue ipfus BCdupla CE. ergo CE minor eft. quàm dux quadragefimx quintx partes ipfius $A$ B. fedCE eft lune diameter, \& AB diftātia, qua cêtrua
lune à vifu noftro diftat. funx igitur diameter, difta
tizequa centrum luneẻa vifu noftro diftat,minor eft

- quàm dux quadragefing quintę partes. Dico etiam
- CE ipfius BA maiorem'ẹfe, quàm trigefimā partē. .

Iun-
for the same reason as before, BC is less than the forty-fifth part of CA, therefore BC is much less than the forty-fifth part of $\mathrm{BA}^{\mathrm{B}}$. And CE is double of BC ; therefore CE is less than two forty-fifth parts of


AB ; but CE is the diameter of the moon and AB is the distance which separates the centre of the moon from our eye. Therefore the diameter of the moon is less than two forty-fifth part of the distance of the centre of the moon from our eye. I say that CE is also greater than the thirtieth part of BA.

## ARIST. DEMAEM,

Tîgãtur enim DE DC. \& cētro quidē $A$, interuallo autem DC circulus defcribatur CDF, atque in eo - aptetur refta linea DF, xqualis ipfi AC. Quoniam - igiturnctus angulus EDC eft \&qualis tecto BCA:


## C

i \& angulus BAC equalis ipf ECD; crit reliquus DE C reliquo HBC equalis: $\%$ triangulum CDE trizngulo $A B C$ aquiangulum. ergo vt $B A$ ad $A C$,ita eft $D$ EC ad CD:\& permurando ve $A B$ ad $C E$, ita $A C$ ad CD; hoc eft ita DF ad CD. Rurfus quoniam angulus E DAC eft vnius recti pars' quadragefsimaquinta, erit circumferentia CD pars centefima a \& oftage....

Let $\mathrm{D} \& \mathrm{E}$ and $\mathrm{B} \& \mathrm{C}$ be joined; after with centre A and distance $\mathrm{DC}^{1}$ let the arc CDF be described; let DF equal to AC be fitted into the same arc. Then, since the right angle EDC is equal to right angle $B C A$ and the angle BAC is equal to $E C D$ itself ${ }^{\mathrm{C}}$, the remaining angle


DEC will be equal to remaining angle HBC. The triangle CED is also equiangular with the triangle ABC , then as BA is to AC , so EC is to CD ; and, by permutando property, as AB is to CE , so is AC to CD, that is, DF to CD. Again, since the angle DAC is the forty-fifth part of a right angle ${ }^{\mathrm{D}}$, then the arc CD will be one hundred and eightieth part of whole

## ETDIST,SOL.ETLVNAE

fima totius circulisecircumferētia DF circuili pars fexta. quare circumferentia CD circumferentix $D$ F trigefima pars èt , atque haber circimferentia 60 , queminor eft circumferentia DF, ad circumfe rentiam DF minorem proportionem, quàm rectali nea $C B$ ad rectam DF. recta igitur linea $C D$ ipfius DF rectę maior eft,quàm trigefima pars. eft autenn DF xqualis AC. ergo DC maior eft, quàm trigefima pars ipfius AC; \& propterea EC ipfius BA maior,erit, quàm trigefima pars. oftenfa eft aút \& mi nor, quàm dux quadragefimx quintę partes ipfius BA.quod oftendendum proponebatur.

$$
\text { FED. COMM } \mathcal{A} N D I N V S .
$$

Itaque conftat ex eo,quod demonftratum eft an- $\mathbf{A}$ gulum BAC dimidij recti effe partem quadr agefima m quintam]Demonftratum eft boc in quarta buius.

Multo igitur minor eft BC, quann quadragefimaB quinta pars ipfius BA] Eft enim $B \mathcal{A}$ maior, quàm $\mathcal{A C}$, . cum maiori angulo fubtendatur.

Et angulus B AC ęqualis ipfi ECDJEx 8. fe xti ele- C mentorum. Quoniam enim ab angulo retto $A C B$ perpendicularis duCta eft CH,funt triangula ACH HCB fimilia to-
 qualis angulo BAC.

Rurfus quoniā anguluṣ D AC eft vnius rectipars. D quadragefima quinta] Hoc demonfratiom ift in quara buiss.

Erit circumferentia CD pars centefima, \& octo- $\mathbf{E}$ sefima totius circuli $]$ Angulus enim rectus confiftit is quarta parte circumferenciae totius circuli, boc eft in grialif. bus nonaginta, cuius circumferentiae pars quadragefima quinta
circle ${ }^{\mathrm{E}}$; the arc DF will also be the sixth part of the circle; thus the arc CD is the thirtieth part of the arc DF . And the arc CD, being less than the arc DF , has to the arc DF itself a ratio less than that which the straight line CD has to the straigt line $\mathrm{DF}^{\mathrm{F}}$.Therefore the straight line CD is greater than the thirtieth part of DF ; but DF is equal to AC , therefore DC is greater than thirtieth part of AC , consequently EC will be greater than the thirtieth part of BA itself ${ }^{\mathrm{G}}$. Then it is proved to be also less than two forty-five parts of BA itself, as we had proposed to demonstrate.

## Federico Commandino

A.Then it is manifest, from what has before been proved, that the angle BAC is the forty-fifth part of half a right angle. This was demonstrated in the fourth proposition of this book.
B. Therefore BC is much less than the forty-fifth part of BA. In fact BA is greater than $A C$ because it subtend a greater angle..
C. And the angle BAC is equal to ECD itself. From $8^{\circ}$ proposition of sixth book of Elements. Indeed, since the perpendicular CH was conduct from the right angle $A C B$, the triangles $A C H$ and $A C B$ become completely similar between them, therefore the angle $B C H$, and cleary $E C D$ is equal to the angle BAC.
D.Again, since the angle DAC is the forty-fifth part of a right angle. This was demostrated in the fourth proposition of this book.
E. Then the arc CD will be one hundred and eightieth part of whole circle. Indeeed a right angle subtend the fourth part of thw whole circle, i.e. ninety degrees, the forty-fifthy part of which arc

## ETST.DFMACKI

¢sinta funt duo gradus, videlicet centefima, tr octogefinat pars totius circuli.

Atque habet circumferentia $C D$, que minor eit circumferentia DF adipfam circumferentiamD

minorem proportionem; quàm reča linea CD ad rectan DF $]$ Nam circuinferentia DF, quae maior of circumferentia . CD ad ipfam C D. circumferentiam maidrim proportionem babet, quam recila linea D F ad re. Caam ${ }^{\prime} C D$; quod demoryfrauit Ptolemgus in principio mia gheie confructionis, quare conuertendo ex 26 quinticitcumferentia $C D$ ad circumferentiann $D F$ minorem bito. bet
are two degrees, or the one hundred and eightyth part of whole circle.
F. And the $\operatorname{arc} \mathrm{CD}$, being less than the $\operatorname{arc} \mathrm{DF}$, has to the arc DF itself

a ratio less than that which the straight line CD has to the straight line DF . Indeed the arc $D F$, which is greater than the arc CD, has to CD itself a ratio greater than that which the straight line DF has to the straight line CD, which Ptolemy has demonstrated at the beginning of Liber Magnae Constructionis; wherefore, by convertendo property, from the proposition 26 of fifthy book, the arc CD has to te arc $D F$

## ETDIT. SOL.ET:LVNAE.

Set proportionem, quim retta linea, CD ad DF reitiom.
Ac propterea EC ipfius BA maior, quàm trigefima pars] Superius namque demoiffratum eft,vt $\mathcal{A B}$ ad $C E$, ita effe $\mathcal{A C}$ ad CD.quare conuertenio vt CE ad $\mathcal{A B}$, ita DC ad C A: Qusod cum DC maior fit, quàm trigefima pars ip

PROPOSITIO XII.

Diameter circuli determinantis in luna opacum, $\boldsymbol{\sigma}^{\beta}$ plendidum diametro luna minor quidem eft, maiorem autem proportionem ba. bet ad ipfam, guàm 89 ad 90.


Sit nolter vifus ad $A$; lunę vero cen trum $B$, quan do conus folem, 8 lunam comprehendens ad vifum coftrum verticem habeat:\& iuncta AB per ipsã pro ducatur planam; quod faciet fectiones, in fphara quidem circulum ; in cono autem rectas lineas. fa-ciat in §phara circulum DEC, \& in cono rectas iv-

A ratio less, than that which the straight line $C D$ to the straight line DF.
G. Consequently EC will be greater than the thirtieth part of BA itself. Indeed we hawe above shown that as $A B$ is to $C E$, so $D C$ is to $C A$. Then, by convertendo property, as $C E$ is to $A B$, so $D C$ is to $C A$. Therefore, being DC greater than the thirtieth part of CA itself, $C E$ will also be greater than the thirtieth part of $A B$.

## PROPOSITION XII

The diameter of the circle which divides the dark and the bright portions in the moon is less than the diameter of the moon, but has to itself a ratio greater than that which 89 has to 90.


Let our eye be at A, while let B be the centre of the moon when the cone comprehending both the sun and the moon has its vertex at our eye; let A\&B joined, and let a plane be carrier through AB; this plane will cut in fact the sphere in a circle while the cone in straight lines. Let it cut the sphere in the circle DEC e the cone the straight lines

## ARIST.DEMAGMOT

meas AD AC CD. ergo CD eft diametercip's culi determinantis in luna opacum \& fplendidum. Dico CD diametro lunç minorem effe, maiorem va 50 ad ipfam proportionem habeře, quàm 89 ad 90 . Itaque CD minorē efié diametro lyna,manifes fum eft.Dico \& maiore habere proportionem, quâ


89 ad 20. Ducatur enim per Bipfi CD paralleat ${ }^{5}$ C, \& $B C$ iungatur. Rurfus eadem ratione erit ang Itus DAC quadragefima quinta pars vnius recti : \& angulus BAC recti. pars nonagefima atque eft BA
C angulus xqualis angulo CBF . ergo \& angulus $C$
A BF eft pars nonagefima recti, videlicet angali $\mathrm{F}_{\mathrm{BE}}$; \& ob id circumferentia CF circumferentix FCE eft nonagefima.quare circumferentia $C E$ ad circumferentiắ EC C câpropostionê hêr, quä 89 ad qo.eftq;

- ipfins CE duplacircumferétia DECiphins veroEC F dupla GEF, ergo DEC circûferétizad circumber
C tiã CEF eam proportionem habebit quam 8 g on habet autem recta linea DCád rectan GF ma jorem proportionem q quam DEC circumferndi
$\mathrm{AD}, \mathrm{AC}, \mathrm{CD}$. Therefore CD is the diameter of the circle which divides the dark and the bright portions in the moon. I say that CD is less than the diameter of the moon, but has to it a ratio greater than that which 89 has to 90 . Indeed is manifest that CD is less than the diameter of the moon. Yet I say that it has to it a ratio greater than

that which 89 has to 90 . Let FG be drawn through B parallel to CD itself, and let $\mathrm{B} \& \mathrm{C}$ joined. Then again, for the same reason as before, the angle DAC will be the forty-fifth part of a right angle: and the angle BAC will be the ninetieth part of a right angle; but the angle BAC is equal to the angle CBF; therefore the angle CBF is also the ninetieth part of a right angle, that is of the angle FBE; therefore the arc CF is the ninetieth part of the $\operatorname{arc} \mathrm{FCE}^{\mathrm{A}}$; so that the arc CE has to the arc ECF that ratio which 89 has to 90 . The arc DEC is double of arc CE and GEF is double of ECF itself, then the arc DEC will have to the arc GEF that same ratio which 89 has to $90^{\mathrm{B}}$. Moreover the straight line DC will have to the straight line GF a ratio greater than that wich the arc DEC has to the arc $\mathrm{GEF}^{\mathrm{C}}$. Therefore the straight line DC will have to the straight line GF a ratio greater than that which 89 has to 90 .


## 

ad circumferentiam GEF . recta igltur linea $D C$ ad sectam GF maiorem proportionem habet, quàre 89 ad 90 .

$$
\text { FED. } \mathcal{C} O M M \mathcal{A} \mathrm{NINP}
$$

Et ob id circumferentia CF circumferentix FCE eft nonagefima ] Anguli enim eundem babet proport :onë quam circumferentiae; in quibusinfiftunt, ex oltma fextiele mentorum.

Ergo DEC circumferentia ad circumferentiam GEF eam proportionem habebit, quàm 89 ad 90.] Ex 15 quinti elcmentaram.

Habet autem reta linea DC ad rectam GF ma$C$ iorem proportionem, quàm DEC circumferentia 2d circumferentiā GEF ] Ex demonftratis à Ptolemes. nam circüferentia GEF ad circüferentiä DEC maiorë̆ babet proportionë, quä GF reita ad reCtă DC.ergo conuertêdo cir cŭferêtia DEC ad circüferêtiư GEF minor Get, quàm reEla DC ad reCZam GF. ideoq́, relta DC ad reEtă GF maiorem proportionem babebit, quam circumferentia D EC ad GEF circumfcrentiam.

```
PROTOSITIO. XIII.
```

Rectal linea fubtendens circumferentiane sirculi, in quoferuntur extrema diametri de terminantis in luna opacum, $\sigma$ /plendidum, qua in terre rombra continetur, maior qui1


## Federico Commandino

A.Therefore the arc CF is the ninetieth part of the arc. Indeed the angles have the same ratio than the arcs which subtend, from the last proposition of the sixth book of Elements.
B.Then the arc DEC will have to the arc GEF the same ratio which 89 has to 90 . From the $15^{\circ}$ proposition of the fifth book of Elements.
C. Moreover the straight line DC will have to the straight line GF a ratio greater than that wich the arc DEC has to the arc GE. By the Ptolemy's demonstrations, indeed the arc GEF has to the arc DEC a ratio greater than that which the straight line GF has to the straight line DC; and therefore the straight line $D C$ is proportionally greater to the straight line GF than the arc DEC is to the arc GEF.

## PROPOSITION XIII

The straight line which subtend the arc of circle, intercepted within the earth's shadow, along which the extremities of the diameter of the circle dividing the dark and the bright portions in the moon move, is less ${ }^{2}$ than the double of the diameter of the moon, but has



## 

rem autem ad ipfarn propartionem ljadet, quàm 89 ad 4s.et minor eft, quàm noma pars diametrifolis, maiorem vero proportionern babet adipfam, quàm 22 ad 22 f. fed ad eam, quae à centro oflis ducitur ad rectos angulos axi, © coni lateribus applicatur, maiorcm babet proportionem, quàm 979 ad IoI2s.

Sit enim folis quidem centrum ad A, terræ vero centrum B,\&lunę centrum C, perfecta exiftente ec clipfi,\& primum tota in terrx vmbram incidente. producaturqúue per ABC planum, quod faciet fectiones in fphęris quidem circulos; in cono autem comprehendente folem \& lunam, rectas lineas. faciat in §phęris maximos carculos DEF GHK LMN. in vmbra vero re rra circulum, in quo feruntur extrema diametri determinantis in luna opacum, \& fplendidum, XLN: \& in cono rectas lineas DGX FKN. axis antem fit ABL. manifeftumeft A L axem contingere circulum LMN : propterea quòd $v m b r a t e r r x$ fit duarum lunarum, $\&$ circumferentia NLX ab axe ABL bifariam fecetur: \& adhucluna primum in terré vmbram incidar. Itaque iungātur XN NL BN LX. ergo LN eft diameter circuli, in luna ppacum,\& fplendidum determinantis: \& BN contingit circulum LMN; quòd B fic ad noftrum vi fum, \& LN diameter circuli determinantis in luna opacum, \& Splendidum. Quoniam igirur XL LN $\varepsilon$ quales funt, duple erunt ipfius LN . quare XN ip- A fius NL minor eft,quàm dupla.iungantur LC CN, B \& LC ad O producatur.multo igitar XN minor eft, C quàin
to it a ratio greater than that which 89 has to 45; moreover it is less than the ninth part of the diameter of the sun, but has to itself a ratio greater than that which 22 hass to 225 . But it has to the straight line drawn from the centre of the sun perpendicularly to the axis and meeting the sides of the cone a ratio greater than that which 979 has to 10125 .

In fact let $A$ be the centre of the sun, let $B$ be the centre of the earth and C the centre of the moon during a total eclipse, before falling completely into the shadow of the earth, and let a plane be carried through ABC, which will cut of course the spheres in circles and the cone comprehending both the sun and the earth in streight lines. Let it cat the spheres in the great circles DEF GHK LMN, while the earth's shadow in the circle XLN, in which the extremities of the diameter of the circle dividing the dark and the bright portions in the moon move, and the cone in the straight lines DGX e FKN. Let after ABL be the axis; then is manifest that the axis ABL touches the circles LMN, because the shadow is twice the moon- breadth, and the arc NLX is bisected by the axis ABL, and moreover that the moon has fallen at first within the estrh's shadow. So let $\mathrm{X} \& N, \mathrm{~N} \&$ $\mathrm{L}, \mathrm{B} \& \mathrm{~N}, \mathrm{~L} \& \mathrm{X}$ be joined; then LN is the diameter of the circle dividing the dark and the bright portions in the moon; BN touches the circle LMN because $B$ is at our eye and $L N$ is the diameter of the circle dividing the dark and the bright portions in the moon. Then, since XL and LN are equal, they will be twice than LN itself; consequently XN is less than double of $\mathrm{NL}^{\mathrm{A}}$. Let L\&C and C\&N be joined, and let LC be produced to O ; then XN is much less than



## 

quàm dupla ipfius LO. Et cum CE perpendicularǐ: D fit ad LB, erit ipfi XN parallela. angulus igitur LX E N eft qqualis angulo CLN. atq; eft NL, $\S q u a l i s L X$, \& LC ipfi CN quare triägulum XNL. fimile eft trià gulo LNC. eft igiturvt XN ad NL, ita NL ad LC. 4.scm fed NLad LC maiorê proportionē hêt,quam 89 ad F 45;hoc eft quadratum ex NL ad quadratum ex LC $C$ maiorem habet proportionem, quā 7921 ad 2025. ergo 8 quadratum ex NX ad quadratū ex NL maiorem proportionem habebit',quam" 7921 ad' 2025 \& ipfa XN ad LO maiorem, quă $7 \cdot \frac{121}{}$ ad 4050 ha- H bet autem 7921 ad 4050 malorem proportionem, $K$ quam 88 ad 45 -quare XN ad LO maiorem propor $\mathrm{L}_{0}$ sionem habebit, quàm 88 ad $45 . \&$ ob id recta linea fubtendens circumferentiam circuli, in quo feruntur extrema diamerri decermingantis in funa opacum \& fplendidum, qux in terree vmbra comprehe ditur, minor eft, quàm dupla diamerri lunx, maiorem autem ad ipfam proportionem habet, quàm 88 ad 45.
lifdem pofitís ducatur à punfoto $A$ ípfi $A B$ 2d re $M$ tos angulos PAR. Dico XN minorem quidé effe, quăm nozam partem diametris folis; maiorem vero ad ipfant proportionē habcre, quàm 22 ad 225 ;
 ad 1125 . Quoniam enim oftenfa eft XN diametri lunx minor, quàm dupla;lunx autem diameter dia $N$ metri folis minor eff, quàm duodeuigefima pars, crit XN ininor, quàm nona pars diamerri folis. Rur fus quoniam XN ad diametrü lunx maiorem próportionem habet, quàm 88 ad $45, \&$ diameter lunae ad folis diamerrum maiorem hêt, quàm 45 ad 900. is.qai $^{2}$ quippe quòd lunx diameter ad diametrü folis ma-
iorem
double of $\mathrm{LO}^{\mathrm{B}}$; and since CL is perpendicular to $\mathrm{LB}^{\mathrm{C}}$, it will be parallel to XN itself ${ }^{\mathrm{D}}$, therefore the angle LXN is equal to the angle $\mathrm{CLN}^{\mathrm{E}}$, and NL is equal to LX and LC to CN itself; therefore the triangle XNL is similar to the triangle LNC, therefore as XN is to NL so NL is to LC ${ }^{4}{ }^{\text {jprop of of sixth }}$ 6. , but NL has to LC a un ratio greater than 89 a $45^{\mathrm{F}}$, that is, the square on NL has to the square on LC a ratio greater than that which 7921 has to $2025^{\text {G }}$. Therefore the square on NX also has to the square on NL a ratio greater than that which 7921 has to 2025, therefore XN itself also has to LO a ratio greater than that which 7921 has to $4050^{\text {H }}$; but 7921 also has to 4050 a ratio greater than that which di 88 has to $45^{\mathrm{K}}$, therefore XN will have to LO a ratio greater than that which 88 has to $45^{\mathrm{L}}$. Therefore the straight line that subtends the arc of circle, intercepted within the earth's shadow, along which the extremities of the diameter of the circle dividing the dark and the bright portions in the moon move, is less than double of the diameter of the moon, but has to it a ratio greater than that which 88 has to 45 .
The same hypotesis being made, let PAR be drawn from the point A perpendicularly to AB itself ${ }^{\mathrm{M}}$. I say that XN is certainly less that the ninth part of the sun's diameter, but has to it a ratio greater than that which 22 has to 225 , and has to PR a ratio greater than that which 979 has to $10125^{3}$. Since it was demonstrated than XN is less than double of the moon's diameter, while the diameter of the moon is less than the eighteenth part of the sun's diameter ${ }^{\mathrm{N}}$, then XN will be less than the ninth part of the diameter of the sun. Again, since XN has to the moon's diameter a ratio greater than that which 88 has to 45 and the moon's diameter has to the sun's diameter a ratio greater than that which 45 has to $900^{15^{5}}$ prop. of fifth b , since the diameter



## 

iorem habeat proportioncm, quàm I ad 20,8 omnia quadragies quinquies fumantur : habebit XN $\mathbf{P}$ ad diametrum folis maiorem proportionem, quàm 88 ad 900 ;hoc eft quàm 22 ad 225 . ducantur a pun y.qurCto B circutúDEF contingentes BYS BVT. \& Y V , YA iungantur.erit igiturve diameter circuli in luna opacum, \& fplendidum determinantis ad diametrum luņ̧, ita YV ad folis diametrum, quòd idé conus folem, \& lunaff comprehendat, ad vifum no frum verticenı habens.diameter autem circuli determinantis in lurta opacum, \& fplendidum ad diametrumallunę maiorem proportionem habet, quàm 89 ad $90 . e \mathrm{rgo} \& \mathrm{YV}$ ad diametrum folis maiorem habet, quàm 89 ad $90: \&$ QY ad YA habebit maio- $S$ rem,quàm 89 ad 90 . Ntautem QYad'YA, ita YA T ad $A S$, cum paraflele fiat $S A$ : YQ . quare \& YA ad $A S$ maiorem habet proportionem, quàm 89 ad 90 . multo igitur YA ad AR maiorem proportionem $V$ habcbit, quàm8 89 ad 90.0 oftenfa eft autem \& XN ad $\mathbf{X}$ diametrum folis maiorem habere proportionem, quàm 22 ad 225 ; \& ex equali.ergo $X N a d P R$ multo maiorem proportionem habet, quàm numerus productus ex 22 , \& 89 ad cum, qui ex $90 \& 225$ pro ducitur.hoc eft 1958 ad 20250: \& horum dimidi2 videlicet 979 ad 10 : 25 .

```
    FED. COMMANDINVS.
```

Quart XN ipflus $N L$ minor eft, quàm dupla] $A$ Sunt enim trianguli $\pm X N$ duo latera XL $L N$ reliquoXN m.iora, ea 20 primi elementorum.

Malto igitur XN minoreft, quàm dupla ipfiusL B
 C diame-
of the moon has to the diameter of the sun a ratio greater than that which 1 has to 20 , if we multilpy throughout by forty five ${ }^{\mathrm{O}}$, then XM will have to the diameter of the sun a ratio geater than that which 88 has to $900^{\mathrm{P}}$ i.e. than that which 22 has to 225 . Let the BYS e BVT be drawn from point $B$ tangent the circle DEF ${ }^{15}{ }^{5}$ prop. of fifth 6 . and letY\&V and Y\&A be joined; then, as the diameter of the circle dividing the dark and the bright portions in the moon is to the diameter of the moon, so YV will be to the diameter of the sun, because the sun and the moon are comprehended by one and the same cone which have its vertex at our eye ${ }^{\mathrm{R}}$ as said before . But the diameter of the circle dividing the dark and the bright portions in the moon has a ratio greater than that which 89 has to 90 , therefore YV also has to the diameter of the sun a ratio greater than that which 89 has to 90 ; QY also will have to YA a ratio greater than that which 89 has to $90^{\mathrm{S}}$. After, as QY is to YA, so is YA to AS, because SA and YQ are parallel ${ }^{\mathrm{T}}$; therefore YA also has to AS a ratio greater than that which 89 has to 90 . Therefore YA will have to AR a ratio much greater than that which 89 has to $90^{\mathrm{V}}$. It is also demonstrated that XN has to diameter of the sun a ratio greater than that which 22 has to 225 ; also XN, by direct proportionality, has to PR a ratio much greater than that which the product of 22 and 89 has to the product of 90 and $225^{\mathrm{x}}$, that is 1958 to 20250 , and clearly also to their half i.e. 979 to 10125 .

## Federico Commandino

A. Consequently XN is less than double of NL: in fact XL ed $L N$ are two sides of the triangle LXN greater than the remaining $X N$, for the $20^{\circ}$ proposition of first book of Elements.
B. Then XN is much less than double of LO: since LO is the moon's diameter it is greater than $L N$ which is the diameter


A.


## 




## ET DIST. SOE ETIVDAE 荘A <br> Chameter circuli,qui in luma opactom, jolendidum deter; minat.

Et cum CL perpëdicularis ad LB] Ex 18 tertij ele $C$ mer:torsu, quad recia linea EL'circulum LMN comtingat.
Erit ipfi XN parallelaj Ex 28 primi elementorŭ, eft.n. D BL ät ad XN perpendicularis, cuin ipfam bifariam feret. s.terii.:

Angulus igitur LXN eft rqualis angulo CLN] E Quoniä.n. LO XN parallehe fumt, erit angulus LNX equa e9.prie lis angulo CLNNSed angilus LXN eft equalis angulo LNX, mi.
 aequales $L C$ CN.ergo đr reliquus angulns $X L N$ ejt aequa lis reliquo LCN, \&s triangulum triangulo fimile.

Sed N Lad L C maiorem proportionem habet, quàm 89 ad 45 JHabet enim $N L$ ad lunae diaqmetrum $L O$ maiorem proportionem, quàm 89 ad 90 ,quod in antecedĕte demonftratum eft.

Hoceft quadratum ex NL ad quadratum exLC 6 maiorem habet proportioné, quàm 7921 ad 2025$]$ Eft enim 7921 numerus quadratus, qui fit ex 89, quadratus, qui ex 4\%.

Et ipfa XN ad LO maiorê, quàm 79 ะ 1 ad 4050] H Nam cum XN ad NL maiorcm ibabeat proportionem, quàm 89 ad 45, boc ef quàm 7921 ad 4005 ; iorem, quam 89 ad 90, hoc eft quam 4005 ad 4050: babe bit ex equali XN ad LO multo maiorem proportionem, quă
 tielementorum.

Habet autem 7921 ad 4050 maiorem propor- $K$ tionem, quàm 88 ad 45 JEft enim 88 ad 45, vt 7921 ad $4050{ }_{80} 8^{\circ}$. fed 7921 ad 4050 maiorem babet propor- 8 .quim tionem, quam ad 4050 4. 48 . ergo 7921 ad 4050 maiorem ti. proportionem babebit, quam 89 ad 45 .

Quare $\mathbf{X N}$ ad LO maiorem proportioné habe- $L$ $G 3$ bit,
of the circle dividing the dark and the bright portions in the moon.
C. And since CL is perpendicular to LB: by the $18^{\circ}$ proposition of tirth book of Elements, because the straight line BL touch the circle LMN.
D. It will be parallel to XN itself: from $28^{\circ}$ proposition of first book of Elements, indeed BL is also at right angle to $X N$, because it cuts into two equal parts.
E. Therefore the angle LXN is equal to the angle CLN: since LO and $X N$ are parallel, the angle $L N X$ will be equal to the angle $C L N^{29^{\circ}}$ prop. of first book. But the angle LXN is equal to the angle LNX, and the angle CNL is equal to a CLN itself ${ }^{5^{\circ} \text { prop.of first booko, since } X L \text { and } L N ~}$ are equal, similary are equal $L C$ and $C N$, therefore the remaining angle $X L N$ is equal to remaining $L C N$ and the triangles are similary, from $3^{\circ}$ proposition of tirth book.
F. But NL has to LC a un ratio greater than 89 a 45: indeed $N L$ will have to the moon's diameter LO a ratio greater than that wich 89 to 90, as we have demonstrated previously.
G. The square on NL has to the square on LC a ratio greater than that which 7921 has to 2025: ideed 7921 is the square of 89, and 2025 is the square of 45 .
H. therefore XN itself also has to LO a ratio greater than that which 7921 has to 4050: since XN has to LO a ratio greater than that wich 89 to 45, i.e. 7921 to 4005, and NL have to LO a ratio greater than that 89 to 90 , i.e. 4005 to $4050, \mathrm{XN}$ will have to $L O$, by direct proporzionality, a ratio much greater than that which 7921 to 4050, for this which we have demonstrated at 13 offifth book of Elements.
K. But 7921 also has to 4050 a ratio greater than that which 88 has to 45: in fact 88 is to 45 as 7921 is to $4050+\frac{45}{88}$, but 7921 has to 4050 a ratio greater than that which $4050+\frac{45}{88}$, therefore 7921 has to 4050 a ratio greater than that wich 89 has to 45 .
L. Therefore XN will have to LO a ratio greater than that which

ARISTDEMACM:



## - - TTET. SOE ET LVNAE. 27

bit, quàm 88 ad 45 I Immo vero longe maios em exain te dictis.

Iifdem pofitis ducaturà puncto $A$ ipfi $A B$ ad re $M$ Ctos angulos PAR 3 Ita vt fecet reftam lineam NKF in puncto P, © rectam lineam XGD in R.

Lunx autem diameter diametri folis, minor eft, N quàm duodénigefima pars]Ex gubuius;folis enim diameter maior eft,quam duodengintrapladiametri lumae.

Et diameter lunx ad folis diametrum maiorem 0 hêt, quã 45 ad 900 . quippe quöd lunx diameter ad diametrū folis maiorem habeat proportionê, quā I ad 20,\& omnia quadragies quinquies furmantur] Ex nona buius.nă cŭ folis diameter minor fit, quàm vigintu pla diametri lmae, babebit diameter lunae ad folis diametrum maioremproportionem, quim 1 ad 20 , hoc ef 45 ad 900 ex 15 quinti.

Habebit XN ad diametrum folis maiorem pro- $P$ portionem, quä 88 ad 900 . ] Immo vero longe maioré.

Ducãtur à puncto $B$ circulum $D E$ contingentes $Q$ BYS BVTjSecent aŭt reltam lineä P AR in punctizs $\mathcal{S} T$.

Erit igitur vt diameter circuli in luna opacum \& $R$ fplendidum determinantis ad diametrom luna, ita YV ad folis diametrum, quod didem conus folem \& Junam comprahendat, ad vifura nofrum vertizent habens. ]ill ${ }_{\text {de }}$ wos hoc lemmate demonfradimess.

Sit nofter vifus ad A, folis centrum B, tiane veron centrum C, quando conus folem \& lunami compre hendensad vifum noftran venti cem haberax. crunt ACB pantta in exdem rectalioeai Bucator pory AC: D planm, quod faciat fectiones, in foherisquiders circukos maximos DfFF; Gftit, in: cono:mwemureGas lineas DGA FKA:iungantaxque BD; CG, \% i punctisD Gad B A ducantur adrectes angulos

88 has to 45: indeed much greater because of the things said before.
M. The same hypotesis being made, let PAR be drawn from the point A perpendicularly to AB itself: so it will cut the straight line NKF at point $P$ and the straight line XGD at $R$.
N . While the diameter of the moon is less than the eighteenth part of the sun's diameter: from $9^{\circ}$ proposition of this book: indeed the diameter of the sun is less than the eighteenth part of the diameter of the moon.
O. And the moon's diameter has to the sun's diameter a ratio greater than that which 45 has to 900 , since the diameter of the moon has to the diameter of the sun a ratio greater than that which 1 has to 20 , if we multilpy throughout by forty five: from ninth proposition of this book, in fact since the diameter of the sun is less than twenty times the moon's diameter, the diameter of the moon will have to the diameter of the sun a ratio greater than that which 1 has to 20 , i.e. 45 to 900 , from $15^{\circ}$ proposition of fifth book.
P . Then XM will have to the diameter of the sun a ratio geater than that which 88 has to 900: to tell the truth, far greater.
Q.Let the BYS e BVT be drawn from point B tangent the circle DEF: these will cut the straight line PAR at the points $S$ and $T$.
R .Then, as the diameter of the circle dividing the dark and the bright portions in the moon is to the diameter of the moon, so YV will be to the diameter of the sun, because the sun and the moon are comprehended by one and the same cone which have its vertex at our eye: we will demonstrate this with this lemma.
Let be our eye at A , the centre of the sun at B , the centre of the moon at C when the cone comprehending both the the sun and the moon has its vertex at our eye. The points A,C,B will be on the same straight line. Let a plane be carried through ACB. This plane will cut the spheres in the great circles DEF e GHK and the cone in the straight lines DGA and FKA; let be joined B\&D and C\&G and let DLF and GMK be drawn from the points $D$ and $G$ at right angles at AB and let DB and GC be produced

## ARIST. DEMAGNIT

DLF GMK:\&DB G Cad püctaN Opro ducantur. Dico vtK Gad GO, ita effe FD ad DN.

Quoniam enim recta linea $A G D$ circulos $D E$ $\boldsymbol{F}$ GHK contingit: 千r a centris $B$ C ad contaClus ducuntur $B D, C G$, 88. ter- erunt anguli $1 D B \mathcal{A} G$
vij. C rettioquare triangulis ABD angulus ADB eft aequalis angulo $\mathcal{A} G C$ triăguli $A C G:$ atque eft ${ }^{\circ}$ angulus DAB vtrique commuoxis . reliquus igitur DBM ef aequalis re liquo GC $\mathcal{A}$. Rurfus triä guli BDL angulus DLB reCtus ef aequalis refto GMC, © angulus DBL eequalis ipfi GCM. ergor © reliquus reliquo aequalis, 貝 triangulum
 triangulo fimile . Vt igitur MGad LD, ita GC I5.qui- ad DB: permutandósp Dt MG ad GC, ita LD ad D8. © cei mit rum dupla,vt KG ad GO,ita FD ad DN. eft autem GK diameter circuli, qui in luna opacum ór $\beta$ plëdidum determinat, GO lunae diameter.ergo po diameter circuli in lusa opa cum, or fplendidum determinantis ad diametrum lurae, $i$ to $E D$ ad $D N, b o c$ eft ad folis diametrumn.
to the points N ed O . I say that as KG is to GO, so FD is to DN .
Since the straight line AGD touches the circles DEF and GHK and both BD and CG are drawn from the centres $B$ and $C$ at contact points, the angles $A D B$ and $A G C$ will be right; therefore the angle $A D B$ of the triangle $A B D$ is equal to the angle $A G C$ of the triangle $A C G$, and the angle $D A B$ is in common, then the remaining DBA is equal to the remaining GCA. Yet the straight angle DLB of the triangle BDL is equal to the straight angle GMC, and the angle DBL is equal to GCM itself; therefore the remaining angle is also equal to the remaining
 angle and the triangle are similar. Then as $M G$ is to $L D$ so $C G$ isto $D B$, and so that, permutando, as MG is to $G C$ so $L D$ is to $D B$ and also their double as KG is to GO so FD is to DN. But GK is the diameter of the circle dividing the dark and the bright portions in the moon, and GO is the diameter of the moon; consequently as the diameter of the circle dividing the dark and the bright portions in the moon is to the diameter of the moon so $F D$ is to $D N$, that is the diameter of the sun.

ETDIT.SOLE ETGVNAE. 2E



## : A ATST. DE M AGMstr.

5 Et QY ad YA habebit maiorem, quàm 89 ad 90] $V t$ enim $\Upsilon V$ ad folis diametrum, ita Q $\mathfrak{Y}$ ad $\Upsilon_{\mathcal{A}}$, cum fint earsm dimidiae, ex 15 quinti.
$T$ Vtautem QY ad YA,ita YA ad AS, cum paralle-29.fri- Ix fintSA YQ Quoniam en:m parallelae funt $S \mathcal{A}$ rD, erit angulus $\gamma \mathcal{A} S$ aequalis angulo $\mathcal{A} \gamma$ Q:atq; ef angulus $\mathcal{A}$ YS reltus aequalis relto $\mathcal{A} \mathcal{Y}$ - ergo ér reliqutss rel. quo $\mathcal{E}$ 4. sexti qualis,et triangulum fimile triangulo.Vt igitur $Q \gamma_{\text {ad }} Y_{\mathcal{A}}$, ita oft $Y_{\mathcal{A}}$ ad $\mathcal{A} S$.
V Multo yitur YAad AR maiorcm proportionem habebit, quàm 89 ad 90 ] Ex 8 quint: eft enim AR mi-
 AR, boc eff folis diameter ad $P R$ maiorem babebit proportiosem, quaim 89 ad 90.
$X$ Ergo XN ad PR multo maiorem proportionem habet, quàm numerus productus ex 22 \& 89 ad eū, qui ex $90 \& 225$ producitur] Quoniam enim XN ad dia metrum folis ma:orem babet proportionem;quă 22 ad 225 . © diameter folis ad PR maiorem babct, quà̀n 89 ad 90 , fiat vt 225 ad 22 ,ita 89 ad alium. erit ad $8 \frac{158}{: 2}$. Cumigitur $X N$ ad diametrum folis maiorem babeat proportionem, quàm 22 ad 25 ,boc eft, quam $8 \frac{15 x}{22}$ ad 89 : © folis diameter ad PR babeat maiorem, quim 89 ad go; babebit ex aequali XN ad TR multo maiorem proportionem, quàn 8 $\frac{15}{2=5}$ ad' 90 . fed $8 \frac{15}{25}$ boc eft $\frac{1!59}{22}-$ ad 90 , eft pt 1958 ad 720250. quodita manifeftum erit. dijpofitis enim numeris in bunc modum, ó decufatim multiplicatis videlicet 225 in 90 .fient $=0250$, 未~ 1 in 1958,fient 1958 .babebit $\frac{12}{25}$ ad $g 0$ eandem proportionem, quam 1958 ad 20250 . quod nos demonftraximus in commentaryis in tertiam propofitioyem. libri Aschimedis de circuli dimenfione, propofitiene feotima - quare XN ad 2R multo maiorcm proportionem babebit, quam numerus
S.QY also will have to YA a ratio greater than that which 89 has to 90 : indeed as YV is to diameter of the sun, so QY is to YA since it is half of those, by the $15^{\circ}$ proposition of fifth book.
T. After, as QY is to YA, so is YA to AS, because SA and YQ are parallel: indeed, since $S A$ and YQ are parallel, the angle YAS will be equal to te angle $A Y Q$, then the right angle $A Y Q$ is equal to the right angle $A Q Y$, therefore the remaining angles will also be equal and both the triangles are similar. As QY is also to YA so YA is to AS.
V.Therefore YA will have to AR a ratio much greater than that which 89 has to 90 : from the $8^{\circ}$ definition of fifth book, in fact AR is less than AS; therefore the double of YA will be also to double of $A R$ itself, that is the diameter of the sun will have to PR a ratio greater than that wich 89 to 90.
X. Also XN, by direct proportionality has to PR a ratio much greater than that which the product of 22 and 89 has to the product of 90 and 225: in fact since $X N$ has to diameter of the a ratio greater than that which 22 to 225, and the diameter of the sun has to PR a ratio greater than that 89 has to 90 , it will occure that as 225 is to 22 so 89 is to something; it will be $8+\frac{158}{225}$. Since $X N$ has also to the diameter of the sun a ratio greater than that which 22 has to 225 , i.e. $8+\frac{158}{225}$ to 89 , and since the diameterof the sun has also to PR a ratio greater than that 89 has to 90 , by direct proportionality, $X N$ will have to $P R$ a ratio much greater than that which $8+\frac{158}{225}$ has to 90 ; but $8+\frac{158}{225} \quad \frac{1958}{225}$ a 90 is as 1958 to 20250; this thing will be so demonstrated: if we have the numbers in this way and multiply at cross that is to say $225 \times 90$ we will have 20250, and 1 for 1958 we will have 1958,

20250

$$
\frac{1958}{225} \quad \frac{90}{1}
$$

$\frac{1958}{225}$ will have to 90 the same ratio wich 1959 has to 20250, which we have demonstrated in the commentarys to tirth proposition of book by Archimedes on measurement of a circle at seventh proposition; therefore $X N$ will be much greater to PR than the number

ETDIST, SOLETLVNAB




Sit eadem figura, quę prius: \& luna ita fe habeat, vt centrum ipfius C fit in axe coni folem, \& terram comprehendentis:maximus autem in Sphxra circulus fit FOM in eodem exiftens plano; \& MO iun gatur.ergo MO eft diameter circuli determinantis in luna opacum,\& fplendidum. Itaque iungãtur $M$ B, BO LX XB MC. rectre igitur linex MB BO contingunt circulum MOP; propterca quòd MO fit diameter circuli determinantis in luna opacum, \& fplendidum. Et quoniam XL eft aqualis $\mathrm{MO}_{\text {; }}$ ytraq; enim ipfarum eft diameter circuli in luna opacum, \& fpleadidum determinantis : erit XML B circumferentia xqualis circumferētix MLO.ideoúf; circumferentia XM ipfi LO requalis . led OL eft $x$ qualis LM.ergo \& XM ipfiML zqualis erit . eft autem \& XB ęqualis BL, quòd punctum B fit terrx cé pofi- trum;habeatq́uc terra puncti, ac cêtri rationem ad toine. fpheqram luna;\& circulus MOP in eodem fit plano.
obtained by moltiplication of $22 \times 89$, i.e. 1958, to that which result by moltiplication $90 \times 225$, or 20250.

## PROPOSITION XIV

The straight line joining the centre of the earth to the centre of the moon has, to the straight line cut offrom the axis towards the moon's centre by the straight line subtending the arc of circle within the earth's shadow, a ratio greater thea that which 675 has to 1 .

For let the same figure as before be consider, and let the moon be so placed that its centre C is on the axis of the cone comprehending both the sun and the earth; let the great circle POM in the sphere (moon's sphere) be in the same extant plane and let be joined M\&O; therefore MO is the diameter of the circle which divides the dark and the bright portions in the moon. Let $\mathrm{M} \& \mathrm{~B}, \mathrm{~B} \& \mathrm{O}, \mathrm{L} \& \mathrm{X}, \mathrm{X} \& \mathrm{~B}, \mathrm{M} \& \mathrm{O}$ be joined, therefore the straight lines MB e BO touch the circle MOP because MO is the diameter of the circle which divides the dark and the bright portions in the moon. And, since XL is equal to MO, both of these are a diameter of the circle which divides the dark and the bright portions in the moon, therefore the arc XML will be equal to the $\operatorname{arc} \mathrm{MLO}^{\mathrm{A}}$, therefore the arc XM is equal to the $\operatorname{arc} \mathrm{LO}$, but OL is equal to $\mathrm{LM}^{\mathrm{B}}$, therefore XM will be also equal to ML ; and consequently XB will be also equal to $\mathrm{BL}^{\mathrm{C}}$ because the point B is the centre of the earth and the earth has the relation of a point and centre to the sphere in which the moon moves and the circle MOPis also in the same plane, therefore BM is perpendicular to XL, ${ }^{\mathrm{D}}$ but

ET DIST. SOL ETEYNAE. 30

A.

CM is also perpendicular to $\mathrm{MB}^{\mathrm{E}}$; therefore CM is parallel to $\mathrm{LX}^{\mathrm{F}}$, but SX is also parallel to MR, therefore the triangle LSX is similar to the triangle MRC ${ }^{G}$; therefore, as SX is to MR so is SL to RC, but SX is less than double of $\mathrm{MR}^{\mathrm{H}}$ and since XN is also less than double of $\mathrm{MO}^{\mathrm{K}}$ then SL is also less than double of CR and $\mathrm{SR}^{4}$ will be much less than double of $\mathrm{RC}^{\mathrm{L}}$. Consequently SC is less than triple of CR , therefore RC will have to CS a ratio greater than that


## ARIST. DE MAGN.

rế proportio nē, quä I ad N. 3. Et quneft vt - BC ad CM,ita MC ad CR,ha
O betq́; BC ad CM maiorem proportione , quā 45 ad r , \&RC ad CS maiorem,quả 1 ad 3: ex $\boldsymbol{x}$ quali MC ad CS maiorem habsbit proportionế, quā 45 ad 3 , hoc cft, quă 15 ad
 1.oftefa eft au tem \& BC ad CM habere maiorem proportionem,
P quàm 45 ad i.rurfus igitur ex xquali BC ad CS ma iorem proportionem habebit, quàm 675 ad I .

$$
\text { FED. } C O M M \mathcal{A} N D I N V S \text {. }
$$

A Erit XML circumferentia eqqualis circumferentix MLO $]$ Ex 28 teriij elementorum. In aequalibus enim circulis aequales rectae lineae aequales circumferentias aufermant.
B Ideoq́ue circumferentia $X M$ ipfi $L O$ xqualis $]$ Quo nimm enim circumferentia XML eft aequalis circumferentie MLO,dempta circumferentia ML ptrique communi; erit .reliqua XM. reliquae LO aequalis.
which 1 has to $3^{\mathrm{M}}$, and since as BC is to CM so is MC to $\mathrm{CR}^{\mathrm{N}}$, then BC has to CM a ratio greater than that which 45 has to $1^{\circ}$ and RC has to CS a ratio greater than that which 1 has to 3; For direct proportionality MC will have also to CS a ratio greater than that which 45 has to 3 i.e. than that 15 has to 1 . Then it was proved that BC has also to CM a ratio greater than that which 45 has to 1 ; again, for direct proportionality, BC has to CS a ratio greater than that which 675 has to $1^{\mathrm{P}}$.


## Federico Commandino

A.The arc XML will be equal to the arc MLO: by $28^{\circ}$ proposition of tirth book of Elements. In fact equal straight lines identify equal arcs on equal circles.
B. therefore the arc XM is equal to the arc LO , but OL is equal to LM : poiché infatti l'arco XML è uguale all'arco MLO, tolto l'arco ML comune ad entrambi, il restante XM sarà uguale al restante LO.

ET: DISTE KOI. HTLUN解.
31
Eft autem \& XB xqualis BL] $\mathcal{A}$ centro enim $E$ ad cir $C$ cumferentiam ducuntur.

Quare BM perpendicularis eft ad XL ] Ex 3 tertï D elementorum, nam reft.r linea BM ex centro ducta circumferentiam $X M L$, ob id reCt mm linean $X L$ bifariam fecat.

Atque eft CM ad MB perpendicularis]Ex i 8 ter- $\boldsymbol{E}$ tü. duifa eft en:m relt. linea ex centro $C$ ad punctum, in quo $B M$ circulum $P O M$ contitugit.
Parallela igitur eft CM ipfi LX J Ex 28 primicle- $\mathbf{F}$ mentorum.

Acpropterea triangulum LXS fimile triangulo $G$ MRC]Namque angulus LXS aequalis eft angulo CMR, © 29 priangulus LSX rectus aequallis retto CRM , ergo o reliquus mi. reliquo aequalis, © or triangulum triangulo fimile.

Sed SX ipfius MR minor eft, quàm dupla] Ex 15 H quinti elementorum.
Quoniam \& XN eft minor,quàm dupla ipfius M K O] Ex demonftratis in antecedente.

EtSR malto minor, quàm dupla iplius RC] Eft L enim RS minor, qnàm SL.

Habebit igirur RC ad CS maiorem proportioné, M quàm 1 ad 3 ]Ex 8 quinti elementorum.
Et quoniam eft vt BC ad CM, ita MC ad CR ] Ex N 4 Jexti nam triangula BMC, MCR fmilia funt ex $\$$ eiufdĕ, quòd ab ang alo reito trianguli BMC ad bafim perpendiculars ducta eft $M$ R.

Habetq̣ue BC ad CM maiorem proportionem, O quàm 45 ad x .]Ex vndecima buius.

Rurlus igitur ex ęquali BC ad CS maiorem pro- $P$ portionem habebit, quàm 675 ad I.] Si enim fiet, vt I ad 45 , ita 15 ad alium, érit ad 675 . It t que qumi.mn $B C$ ad CM maiorem proportionë babet, quadm 45 ad I , boc eff, quam 675 ad 15 ;'ّ̛ MC ad CS maiorem,quim 15 ad 1 , ba bebit
C. And consequently XB will be also equal to BL : indeed both of them are drawn from center $B$ to the circle.
D. Therefore BM is perpendicular to XL: from $3^{\circ}$ proposition of tirth book of Elements, indeed the straight line BM is drawn from the center to the circle XML and for this reason per questo bisect the straight line XL.
E. But CM is also perpendicular to MB : from $18^{\circ}$ proposition of tirth book, indeed a straight line is drawn from the center $C$ to point in which BM touch the circle POM.
F. Therefore CM is parallel to LXQuindi CM è parallela ad LX: from $28^{\circ}$ proposition of first book of Elements.
G.Therefore the triangle LSX is similar to the triangle MRC: indeed the angle LXS is equal to the angle CMR, and the right angle LSX is equal to the straight angle CRM. Therefore the remaining angles will be also equal, and the triangles will be similar.
H.But SX is less than double of: from $15^{\circ}$ proposition of fifth book of Elements.
K. And since XN is also less than double of MO: as shown previously.
L. SR will be much less than double of RCS: in fact RS is less than $S L$.
M. therefore RC will have to CS a ratio greater than that which 1 has to 3: from $8^{\circ}$ proposition of fifth book of Elements.
N. Since as BC is to CM so is MC to CR: from $4^{\circ}$ proposition of sixth book, indeed the triangle $B C M$ and $M C R$ are similar by $8^{\circ}$ proposition of the same, because the perpendicular line is drawn from the straight angle of the triangle BMC to the base MR.
O. Then BC has to CM a ratio greater than that which 45 has to 1 : from the eleventh proposition of this book.
P. Again, for direct proportionality, BC has to CS a ratio greater than that which 675 has to 1: if be it as 1 is to 45, so 15 will be to another, which will be 675. And so, since BC has to CM a ratio greater than that 45 has to 1, i.e. 675 to 15, MC will have to CS, for direct proportionality, a ratio greater than that 15 has to 1 .



## - EAT DIST: St)L:TET EVNAE. A.g3

bebit ex aequali BC ad CS maiorem proportionem, quiw 675 ad .

> PROTOSITIOXV.

Solis diameter ad diamnetrum terre maiorem babet proportionem, quàm 19 ad $3 ; \mathrm{min}_{\mathrm{g}}$ norem rvero, quàm 43 ad $\sigma$.

Sit enim folis quidem centrum $A$, terrx vero cen trum B,\& lunx centrum C, perfecta exiftente ecclifi, hoc eft ita vt puncta $A B C$ in eadem recta linea có ftituantur: \& per axem producatur planum, quod faciat fectiones, in fole quidem circulŭ DEF ; in terra vero circulum GHK, \& in vmbra circumferentiä NX; denique in cono rectas lineas DM FM. iungaturq́ue NX, \& per punctum A ducatur ipfi AM ad rectos angulos OAP. Quoniā igitur NX minor eft, A quàm nona pars diametri folis; habebit OP ad NX multo maiorem proportionem, quam 9 ad $1: \&$ per conucrfonem rationis MA ad AR minorem proportionem habebit, quàm 9 ad 8. Rurfus quoniam $C$ AB ipfius BC maior eft, quàm duodeuigintupla, crit multo maior, qual duodeuigintupla ipfius $B R . D$ ergo $A B$ ad $B R$ maiorem proportionem habet, quaz $183 \mathrm{~d} \mathrm{I}: \&$ conuertendo RB ad BA minorem, quàm $26 . q$ wi 1 ad I8:componendoque RA ad AB minorem ha- ti . bet, quàm 19 ad 18 . oftenfa eft autem \& MA ad AR minorem habere proportionem, quàm 9 ad 8. ergo 28.qu ex xquali MA ad AB minorem habebit proportionem, quä 171 ad 144: \& quā 19 ad 16, partes enim codem modo multiplicium eandem habent proportionem
per proporzionalità diretta $B C$ rispetto a $C S$ avrà un rapporto maggiore di 675 ad 1.

## PROPOSITION XV

The diameter of the sun has to the diameter of the earth a ratio greater than that which 19 has to 3, but less than that which 43 has to 6.

Let A be now the centre of the sun, while B the centre of the earth and C the centre of the moon when the eclipse is total, so that the points ABC may set in a straight line, and a plane be carried throug the axis which will cut the sun in the circle DEF, the earth in GHK and the shadow in the arc NX, and at last the cone in the straight line DM and DF; let N\&X be joined and from point A let OAP be drawn perpendicularly to AM itself. Then, since NX is less than ninth part of the diameter of the sun, therefore OP will have to NX a ratio much greater than that which 9 has to $1^{\mathrm{A}}$ and, convertendo, MA will have to $A R$ a ratio less than that wich 9 has to $8^{B}$. Again, since $A B$ is greater than 18 times $\mathrm{BC}^{\mathrm{C}}$, then it is much greater than eighteen times $\mathrm{BR}^{\mathrm{D}}$, therefore AB has to BR a ratio greater than that which 18 has to $1^{26^{\circ}}$ prop. of fifth b , and, for inverse proportionality, RB has to BA a ratio less than that which 1 has 18 and, componendo, RA has to AB a un ratio less than that which 19 has to 18 . Then it was proved that MA also has to AR a ratio less than that which 9 has to $8 .{ }^{28^{\circ}}$ prop. of fifth b. Therefore, ex aequali, MA will have to AB a ratio less than that which 171 has to $144^{\mathrm{E}}$ and than that which 19 has to 16 , because parts have the same ratio as the same multiplies of them.



## 

portionem. quare per conuerfonem rationis $A B$ se. quaf ad BM maiorem proportionem habet, quàm 19 ad i: 3. vt autë $A M$ ad MB, ita DEF circuli diameter ad F diametrum circuli GHK. folis igitur d.anieter ad diametrum terra maiorem habet proportionem, quìm 19 ad 3. Dico prxterca minoré proportioné habere,quàm 43 ad 6 . Quoniam enim BC ad CR C maiorem habct proportionem, quàm 675 ad I , habebit per comuerfionem rationis CB ad BR propor $30 . q 40$ tionem minorem, quàm 675 ad 674 . fed AB ad BC . minorem proportionem habet, quàm 20 ad i.ergo ex xquali AB ad BR minorem habebit proportionem, quàm r 3500 ad 674 , hoc eft quä 6750 ad $337^{\circ}$ \& conucrtendo, cóponendoq̣uc R त ad AB maiorem proportionem habebit, quàm 7087 ad 6750 . th Quòd cum NX ad OP maiorem habeat proportionem, quàm 979 ad roi 25 , habebit conuertēdo $O P$ ad NX minorem proportionem, quàm 10125 ad 979. Vt autem OP ad NX, ita AM ad MR . ergo \& A ti MadMR minorem proportionem habebit, quàm $M$ 10125 ad 979:\& per conuerfionem rationis MA ad AR habcbit maiorem proportionem quàm 10125 ad 9146.fed RA ad AB maiorem proportionem ha ber, quàm $7087 \mathrm{ad} 6750 . e x x q u a l i$ igitur MA ad A N B maiorē habebıt proportioné, quã numerusprodu Ctus ex ion $25^{\circ}$ \& 7087 ad eúqui ex $9 \mathrm{r} 46,86750$ productur; hoceft quàm 7.1755875 ad 61735900. habet autem \& 71755875 ad 6173550 . maiorem. proportionem, quàm 43 ad 37. ergo $\& M A$ ad $A B$ maiorem phoportionem habebit, quàm 43 ad 37. \& per conuerfionem rationis AM ad MB habtbit minorem proportionem, quàm 43 ad" 6 . fed vt AM adMB, ita cft folis diameter ad diametrum tertx.

Therefore, convertendo, ${ }^{30}$ prop. of fitth b. AB has to BM a ratio greater than that 19 has to 3 , but as AM is toMB, so is the diameter of the circle DEF is to the diameter of the circle $\mathrm{GHK}^{\mathrm{F}}$; therefore the diameter of the sun has to the diameter of the earth a ratio greater than that which 19 has to 3 . Moreover I say that it has to it a ratio less than that which 43 has to. Indeed since $B C$ has to $C R$ a ratio greater than that which 675 has to $1^{\mathrm{G}}$ therefore, convertendo, CB will have to $\mathrm{BR}^{30^{\circ}}$ prop. of fitith b. a ratio less than that which 675 has to 674 , but AB has to BC a ratio less than that which 20 has to $1^{\mathrm{H}}$. Therefore, ex aequali, $A B$ will have to $B R$ a ratio less than that which 13500 has to 674 , that is, than that which 6750 has to $337^{\mathrm{K}}$, and for inverse poporzionality and, componendo, ${ }^{26^{\circ}}$ prop. of fifth b. RA will have to AB a ratio greater than that which 7087 has to 6750 . Therefore since NX has to OP a ratio greater than that which 979 has to $10125,{ }^{\text {L } 188^{\circ}}$ prop. of fifth b. then, for inverse proporzionality, OP will have to NX a ratio less than that which 10125 has to 979 . Then, as OP is to NX, so is $A M$ to $\mathrm{MR}^{\mathrm{M}}$, therefore AM also will have to MR a ratio greater than taht which 10125 has to 979 and therefore, inversely, MA will have to AR a ratio greater than that which 10125 has to 9146 ; but RA has to AB aratio greater than that which 7087 has to 6750 , therefore, ex aequali, MA will have to AB a ratio greater than that which the product of 10125 and 7087 has to the product of 9146 and 6750 , that is, 71.755 .875 to $61.735 .500^{\mathrm{N}}$; but 71.755 .875 has also to 61.735 .500 a ratio greater than that which 43 has to $37^{\circ}$, therefore MA will have to AB a ratio greater than thath which 43 has to 37 and, convertendo, AM will have to AB a ratio greater than that which 43 has to 6 . But, as AM is to AB , so is the diameter of the sun to the diameter of the earth;



## 

ergo diameter folis ad terrx diametrum minorem proportionem habebit, quàm 43 ad 6 . oftenfa eft au tem \& maiorem habere prẹportioné, quā 19 ad 3.

## FED. $\subset O M M \mathcal{A} N D I N V$.

Quoniam igitur NX miuor eff, quàm nona pars A diametri folis, habebit OP ad NX multo maiorem proportionem, quàm 9 ad I.] Ex 12 buius.ex quo fequitur ex 8 quinti $N \mathrm{X}$ ad diametrum folis minorem babere proportionum quamz I ad 9.quare conuertendo ex 26 quinvi diameter folis ad NX maiorem babet proportionem, qua 9 ad I. ©́ OP quac maior eft,quàm folis diameter, ad NX multo inaiorem proportionem babet, quam 9 ad x .fed $\mathrm{vt}, \mathcal{A}$ 0 ad RN, hoc eft vt earum duple OPad NX, ita erit $\mathscr{A M}$ ad MR obfimilitudinem triangulorim AMO RMN. ergo $\mathcal{A} M$ ad MR multo maiorem proportionem babebit,quam 9 ad I .

Et per conurrfionem rationts MA ad AR mino- B rem proportiouẽ habebit, quā 9 ad 8] Ex 30 quinti.

Rurfus quoniam AB ipfins BC maior eft, quàm C duodeuigintuplajEx 7 .huius.

Erit multo maior, quam duodeuigintupla ipfins $D$ BRJEf' enim BR minor, quadim BC.

Ergo ex $\varepsilon q u a k i$ MA ad AB minorem habebit pro $E$ portionem quàm 171 ad 144,7 _uoniam enim $M \mathcal{A}$ ad AR minorem propoftioniem, babet,quain 9 ad 8 , boc eft, qua corum videugintupla, videlicet 171 ad 152 : babet autem RS ad $\mathcal{A B}$ proportionem minorem, quàm 19 ad I 8 . fiat 2t 19 ad 18 ,ita 152 ad alium; erit ad 144 . Cuisitur MAs ad $A \mathrm{AR}$ minorem babeat proportionem, quim 171 ad 152 ; bageatóg R $\mathcal{A}$ ad $\mathcal{A B}$ proporttonem minarem, quàm $1 \leqslant 2$ ad 44: ex aequali MiAd AB misortm proportionem babe$I 2$ bit,

Therefore the diameter of the sun has to diameter of the earth a ratio less than that which 43 has to 6 ; it was also proved that it has to it a ratio greater than that which 19 has to 3 .

## FEDERICO COMMANDINO

A.Then, since NX is less than ninth part of the diameter of the sun, therefore OP will have to NX a ratio much greater than that which 9 has to 1 . For the $12^{\circ}$ proposition of this book, from this it follows, by $8^{\circ}$ proposition of fifth book, that NX has a ratio to the diameter of the sun less than that which 1 has to 9; therefore, convertendo, from $26^{\circ}$ proposition of fifth book, the diameter of the sun has to $N X$ a ratio greater than that which 9 has to 1 , and $O P$, which is greater than the diameter of the sun, ${ }^{8^{\circ}}$ prop.of fifth b. to NX has a ratio much greater than that 9 has to 1; but, as $A O$ is to RN, or as OP is to NX, which are their double, ${ }^{15^{\circ}}$ prop.of fifth $b$. so will be $A M$ to $M R$ for similarity of the triangles $A M O$ and $R M N$; therefore $A M$ to $M R$ will have also a ratio much greater than that 9 has to 1.
B. And, convertendo, MA will have to AR a ratio less than that wich 9 has to 8 . By the $30^{\circ}$ proposition delof fifth book.
C. Again, since AB is greater than 18 times BC . By $7^{\circ}$ proposition of this book.
D.Then it is much greater than eighteen times BR. In fact BR is much less than BC.
E. Therefore, ex aequali, MA will have to AB a ratio less than that which 171 has to 144 . Indeed since MA has to AR a ratio less than that which 9 has to 8, that is than their nineteenth part, that is to say than that which 171 has to 152: therefore $R A$ has to $A B$ a ratio less than that which 19 has to $18 ; 19$ is to 18 so as 152 will be to another number which will be 144. Therefore since MA has to AR a ratio greater than that which 171 has to 152, and since RA has to AB a ratio less than that which 152 has to 144 , ex aequali MA will have to $A B$ a ratio less



## ETODSTOOR: ETEVNAER 3

bit, quam 171 ad 144;boc eft quàm 19 ad 16.
Vt autem AM ad MB, ita DEF circuli diameter ad diametrum circuli GHK.]Iumgantur AD BG. erit trianguli 1 AD $\mathcal{A}$ angulus $\mathcal{A D M}$ reCIg aequalis reCfo BGM trianguli $M G B$. Sed angulus $D M \mathcal{A}$ eft commmis verique. ergo é reliquus reliquo aequalis; © triangulum triangulo fimile: Vt igitur $\mathcal{A M}$ ad $M B$, ita $\mathcal{A D}$ ad $B G$, ór ita earum duplae, videlicet diameter circuli DEF ad circuli G H K diametrum.

Quoniam enim.BC ad CR maiorem habet proportionem, quàm 675 ad i. ]Ex 13 buius.

Sed AB ad B C minorem proportionem habet, quàm 20 ad 1$] E x 7$ buius.
Ergo ex æquali' AB ad BR minorem habebit pro portionê, quãm 13500 ad 674 ,hocteft quàm 6750 2d 337] Nam cum AB ad BC miniorem babeat proporionem, quàm 20 ad I , boc eft quim 13500 ad 675 , $\sigma$ CB ad $B R$ babeat minorem proportionem, quadm 675 ad 674; babebit ex aaquali $A B$ ad BR minorem proportionem, quims 13500 ad 674 , boc ef, quàm eorü dimidia 6750 ad 337.

Quòd cum NX ad OP maiorem habeat propor- L tionem, quàm 979 ad 10125$] E x$ I 2 bunss.

Vtautem OP ad NX, ita AM ad MR] Sunt enim $M$ -riangula $\langle M O$ RMN inter fe fimaliat, ut fupenjus dictū̆ eft.

Ex áquali igitur MA ạd AB maioré hahebit pro $N$ portionem, quàm numerus productus ex io $\overline{\mathrm{T}} 25$ \& 7087 ad eū,qui ex $9: 146 \$ 6750$ producitur'hoc eft,
 AR maiorem babet proportionem, quam 1012.5 ad 9146 , \& R $\mathcal{A}$ ad' $\mathcal{A B}$ babet maiorem, quam 7087 ad 6750 , fat
 fi tnim multiplicemus 10125 per 7087, お quod pkoducitur, videlicet 71755875 diuidamus per 9146 , ekibunt

Than that which 171 has to 144; i.e. than that 19 has to 16.
F. But as AM is to MB , so is the diameter of the circle DEF is to the diameter of the circle. Let be joined $A D \& B G$, the right angle $A D M$ of the triangle MDA will be equal to the right angle BGM of the triangle MGB, but the angle DMA is common to both triangles, therefore the remaining angle is equal and the two triangles are similar. Then as $A M$ is to $M B$ so $A D$ is to $B G$, and so their double also, or the diameter of the circle DEF to diameter of the circle GHK.
G. Indeed since BC has to CR a ratio greater than that which 675 has to 1. From the $13^{\circ}$ proposition of this book.
H.But AB has to BC a ratio less than that which 20 has to 1 . Dalla $7^{\circ}$ proposizione di questo libro.
K.Therefore, ex aequali, AB will have to BR a ratio less than that which 13500 has to 674 , that is, than that which 6750 has to 337 . Indeed since $A B$ has to $B C$ a ratio less than that whici 20 has to 1 , that is which 13500 has to 675 and $C B$ rispetto a $B R$ un rapporto minore di 675 a 674 , per proporzionalità diretta $A B$ avrà rispetto a $B R$ un rapporto minore di 13500 a 674 , cioè della loro metà 6750 a 337.
L. Therefore since NX has to OP a ratio greater than that which 979 has to 10125 . From $12^{\circ}$ proposition of this book.
M. Then, as OP is to NX, so is AM to MRCome allora OP sta ad NX, così AM sta ad MR. Indeed the triangles AMO and RMN are similar to each other as previously mentioned.
N.therefore, ex aequali, MA will have to AB a ratio greater than that which the product of 10125 and 7087 has to the product of 9146 and 6750, that is, 71.755 .875 to 61.735 .500 . Indeed since MA has to $A R$ a ratio greater than that which 10125 has to 9146, and RA to $A B$ has a ratio greater than that which 7087 has to 6750, 9146 will be to 10125 , so as 7087 will be to another number which will be $7845+\frac{5505}{9146}$; indeed if we multiply 10125 and 7087 and we divide their product, i.e. 71.755 .875 by 9146, we will have



## ET DIST: SOL: ET LVNAE. 136

$7845 \frac{15}{92} 45$. Itaque cum $M \mathcal{A}$ ad $\mathcal{A R}$, maiorcm babeat pro
 ad 7085; © R A ad AB babeat maiorem, quam 7087 a.t 6750: babebit ex equali MAA ad $\mathcal{A B}$ miorem proportionem, quàm $7845 \frac{5155}{944}$ ad 6750 . Sed $7845 \frac{5105}{9.46}$ boc eft $\frac{71758755}{7176}$ ad 6750 eft ut 71755875 ad 61735500 . quo.l quidé numeris decufatim multiplicatis perfpicuum erit ; ex
${ }^{6} 773500$ ïs, que nas demonftrauimus in commers-
 cbimedis de circuli dimenfione, propofitione feptima, $\nu t$ pror xime diximus.ergo $M \mathcal{A}$ ad $\mathcal{A} B$ maiorem babet proportiomem, quam numerus produitus ex 10125 et 7087 ad eimas qui ex 9146 © 6750 producitur.

Habetautem \& 71755875 ad 61735500 maiorem proportionem, quàm 43 ad 37.] Sienim fiat pt 0 43 ad 37, ita 71755875 ad aliul. erit ad 61743427 qui maior eft, quä 6:735500,ergo. 71755875 ad 61735500 maiorem babebit proportionem, quam ad 61743427, boc 8 quieft, quim 43 ad 37.

PROPOSITIO.XVI.

Sol ad terram maiorem quidem proportionem babet, quàm 6859 ad 27 , minorers vero, quàm 79507 ad 216.


## Sit enim folis quidem diameter $A$, terra vero dia <br> metex

$7845+\frac{5505}{9146}$. So since MA has to AR a ratio greater than that which 10125 has to 9146, that is which $7845+\frac{5505}{9146}$ has to 7085, and since $R A$ has to $A B$ a ratio greater than that which 7087 has to 6750 , MA will have, ex eaquali, to $A B$ a ratio greater than that which $7845+\frac{5505}{9146}$ has to 6750 . But $7845+\frac{5505}{9146}$, that is $\frac{71755875}{9146}$, is to 6750 as 71.755 .875 is to 61.735 .500 , which is certainly clear by multiplying the numbers on cross,
61.735 .500

71755875

according to what we have shown in the commentaries to the third proposition on the measure of the circle, seventh proposition, as we have said just before. Therefore MA has to $A B$ a ratio greater than the product of 10125 and 7087 has to the product of 9146 and 6750.
O.But 71.755 .875 has also to 61.735 .500 a ratio greater than that which 43 has to 37 . Indeed if it was as 43 is to 37 so 71.755 .875 will be to another number, i.e. will be to 61.743.427, which is greater than 61.735.500, therefore 71.755 .875 will have also to 61.735 .500 a ratio greater than which to 61.743.427, that is than that which 43 has to 37.

## PROPOSIZIONE XVI

The sun has to the earth a ratio greater than that which 6859 has to 27, but also less than that which 79507 has to 216.


Let A be the diameter of the sun, while B the diameter

# $\therefore$ ARIST. DEMA:CNIT 


*- meter B.demonftratum iam eft, vt folis fphixa ad terrx finhram, ita cffe cubum, quifit ex diametro folis ad cubum, qui ex diametro terré, quemadmodum \& in luna.ergovt cubus ex A ad cubum ex $\mathrm{B}_{\text {. }}$ ita fol eft ad rerram. cubus autem ex A ad cubum cx $B$ malorem proportionem hâbet, quàm 6859 ad 27 ;m norem vero, quàm 79507 ad 216 ; etenim $A$ ad B maiorem habet proportionem, quàm 19 ad 3 , minorem vero, quàm 43 ad 6 . Quare \& fol ad teerrã maioré proportionem habebit, quàm 6859 ad 27 ; minorein vero, quàm $79507 \mathrm{ad}=16$.

## FED. $\quad$ COMM $A N D I N \not \subset$ S....,

Demonftratumiam eft, vt folis fphara ad terrx fpherram, lta effe cubum, qui fit ex diametro folis ad cubum, qui ex diametro terre, quemadmodam \& in luna In decima enim propofitione buius demonftratum if ot cubus quift ex diametro folis at cubum qui ex diaǹe tro luuae, ita effe fpheravasolis ad lance JRherame qued fombliter in terra demonfirabitur.

TROTOSITIOXVII.
Diameter terves at diametrum lune in maioriquidem ef proportione quàm Io\& ad

## A

B

* of the earth. It is already proved that, as the sphere of the sun is to the sphere of the earth, so is the cube on the sun diameter to the cube on the eart's diameter, exactly as for the moon; then, as the cube on A is to the cube on B , so is the sun to the earth, therefore the cube on A to the cube on B has a ratio greater than that which 6859 has to 27 , but less than that which 79507 has to 216 ; indeed A has to B a ratio greater than that which 19 has to 3, but less than that which 43 has to 6 . Therefore the sun will have also to the earth a ratio greater than that which 6859 has to 27 , but less than that which 79507 has to 216 .


## Federico Commandino

* It is previously proved that, as the sphere of the sun is to the sphere of the eart, so the cube on the sun diameter is to the cube on the earth diameter, just s for the moon. Indeed it is proved in the tenth proposition of this book that as the cube on the diameter of the sun is to the cube on the diameter of the moon, so the sphere of the sun is to the sphere of the moon, which similarly we have shown for the earth.


## PROPOSIZIONE XVII

The diameter of the earth is to diameter of the moon in a ratio greater than that which 108 has to 43, but less than that which

## ETDIST. SOLETEVNAE. 37

Fa, in minorivero, quàm 60 ad 19 .
Sit folis qnidem dıameter $A$, lunx diameter $B$; eterrx vero C.Et quoniam A ad C. minorem propot tionem habet, quàm 43 ad 6 , habebit conuertendo 5


Cad A maiorem proportionem, quàın 6 ad 43 . fed A ad B maiorem proportionem habet, quàm is ad r.ergo ex ${ }^{2}$ quali $C$ ad $B$ maiorem habebit propor ${ }^{2}$ tionem, quàm 108 ad 43 . Rurfus quoniam $A$ ad $C$ maioremproportionem habet, quàm 19 ad 3 , conuertendo C ad A minorem habebit, quàm 3 ad 19. habet autem $A$ ad' $B$ minorem proportionem, qua zoad i.ex requali igitur $C$ ad $B$ minorem proportio nè habebit, quàm 60 ad 19.

$$
F E D . \quad C O M M \mathcal{A} N D I N V S
$$

$\qquad$

Et quoniam Aad C minorem proportionem habet, quàm 43 ad 6 EEx 14 buius.

Sed A ad B maiorem proportionem habet, quā I8 ad I $]$ Ex g, buius.

Ergo ex æquali C ad B maiorem proportionem, habebit, quàm 108 ad 43] Quoniam enim $C$ ad $\mathcal{A}$ maiorembabet proportionem quim 6 an $43: \sigma \mathcal{A}$ ad $B$ maiorës:; quim 18 ad 1 , fat vt 18 ad 1 , ita 43 ad alium. erit ad 2 霍 cum igitur $C$ ad $\mathcal{A}$ maiorem proportionem babeat, quàm 6'


60 has to 19.
Let now A be the diameter of the sun, let B be the diameter of the moon, while C that of the earth. Since A has to C a ratio less than that which 43 has to $6^{\mathrm{A}}$, therefore, inversely, C will have to A

A
B
C

A ratio greater than that which 6 has to 43 . But A also has to Ba ratio greater than that which 18 has to $1^{\mathrm{B}}$, therefore, ex aequali, C will have to B a ratio greater than that which 108 has to $43^{C}$. Again, since A has to C a ratio greater than that which 19 has to $3{ }^{\text {D }}$, for inverse proportionality, C will have to A than that which 3 has to 19 , then A to B has a ratio less than that which 20 has to $1,{ }^{\mathrm{E}}$ and for direct proportionality, also C will have to B a ratio greater than that 60 has to $19^{F}$.

## Federico Commandino

A. Since A has to C a ratio less than that which 43 has to 6 . From $14^{\circ}$ propositionof this book.
B. But A also has to B a ratio greater than that which 18 has to 1. From $9^{\circ}$ propositionof this book.
C. therefore, ex aequali, C will have to B a ratio greater than that which 108 has to. Indeed, since $C$ has to $A$ a ratio greater thatn that which 6 has to 43 and $A$ has to $B$ a ratio less than that which 18 has to 1 , let it is as 18 is to 1 , so 43 is to another unknow which will be $2+\frac{7}{18}$, since $C$ to $A$ has a ratio greater than that which 6 has to 43 and $A$ has to $B$ a ratio greater than that which 43 has to $2+\frac{7}{18}$, $A$ will have to $B$, by direct proportionality,

## ARIST. DEMAGN

li A ad $B$ maiorcm proportionem quam 6 ad $1 \frac{7}{18}$ boc tef, quim 108 ad 43 , quod numeris decuffation 108 multiplicatis manifefte conflat, ex $\ddot{\text { üs,qua }}$, fupe- $\frac{6}{1} \frac{43}{28}$. rius dicta funt.
D Rurfus quoniam A ad C maiorem proportionế habet, quàm 19 ad 3 JEx 14 buius.
E. Habet autem A ad B minorem proportionem, quàm 20 ad I $]$ Ex g.buius.
F Ex xqualigitur C ad B minorem proportioné. habebit, quā 60 ad 19]Fiat $\partial \boldsymbol{\text { ot }} 20$ ad I, ita 19 ad alium. erit ad ${ }_{20}^{19}$. quare cum Cad $\mathcal{A}$ miuorem proportionem babeat,quäm 3 ad 19 , z $\mathcal{A}$ ad $B$ minorem, quam 19 ad $\frac{9}{20}$. ex equali C ad B minorem babebit proportio- so uem, quarm 3 ad $\frac{3 .}{20}$ hoc eft, quam 60 ad 19 . $\frac{3}{1}-\frac{19}{20}$
PROPOSITIO. XVIII.

Terra ad lunam in maiori quidern ef proportione, quam 1259712 ad79507, in minori vero,quàm 216000 ad 6859.


Sit enim terræ diameter A , lunę vero B. quare $\mathbf{A}$ ad $B$ maiorem quidem proportionem habet, quàm 108 ad 43 , mi norem vero, quàm 60 ad 19 . ergo $\&$ qui fit ex A cubus ad cubum qui ex B maiorem pro portionembabet,gnàm 1259712 ad 79507 ,minoré
vero

A ratio greater than that which 6 has to $2+\frac{7}{18}$,that is, which 108 has to 43, as clearly seen 108
by multiplication in cross in croce the numbers according to what was said before $\quad \frac{6}{1} \quad \frac{43}{18}$
D. Again, since A has to C a ratio greater than that which 19 has to 3 . From $14^{\circ}$ proposition of this book.
E. Then A to B has a ratio less than that which 20 has to 1 . From $9^{\circ}$ propositionof this book.
F. And for direct proportionality, also C will have to B a ratio greater than that 60 has to 19. As 20 is to 119 will be to another number wich will be $\frac{19}{20}$, therefore, since $C$ has to $A$ a ratio greater than that a ratio less than that which 3 has to 19 and $A$ has to $B$ less than thati 19 has to $\frac{19}{20}$,

60
By direct proportionality $C$ will have $\frac{3}{1} \quad \frac{19}{20}$
to $B$ a ratio less than that which 3 has to $\frac{19}{20}$ i.e. which 60 to 19 .

## PROPOSIZIONE XVIII

The earth has to the moon a ratio certainly greater than that which 1.259 .712 has to 79.507, but surely less than that which 216.000 has to 6859. ${ }^{5}$

## A

## B

Indeed let Athe diameter of the earth, while B that of tha moon; therefore A has to B a ratio greater than that 108 has to 43 , but less than that which 60 has to 19 ; therefore also the cube on $A$ has to the cube on di B a ratio greater than that which 1259712 has 79507 , but less than that which

ET DIST. SOL. ET LYNAE. 3番 veroxquàm 216000 ad 6859 . Sed vt cubus ex A ad cubum ex $B$, ita eft terra ad lunā.terra igitur ad la mam maiorem quidem proportionem habet, quàm 72597az ad $79507 x$ minorem vero, quàm 316000 ad 6859.

## FINIS.

## $\boldsymbol{P} \quad \mathrm{S} \cdot \mathrm{A} \quad \mathrm{V}$ R I. <br> Apud Camillum Francifhinum. $M D L X X I$.

216000 has to 6859 . But as the cube on A is to the cube on B , so is the earth to the moon, therefore the earth has to the moon a ratio greater than that which 1259712 has to 795071 , bat yet less than that which 216000 has to 6859 .

The End

## PESARO

By Camillum Franceschinum
M D L X X I I

## NOTE

${ }^{1}$ We mean the distance AC , as is reported in the manuscript of the Cod.Vat.Gr. 204 and Vat.Gr. 191
${ }^{2}$ In the text maior. Obvious error.
${ }^{3}$ In the latin text quam 379 ad 1125 . Obvious error.
${ }^{4} \mathrm{~S}$ is missing in the latin text.
${ }^{5}$ Numbers completely different from those of the next time translators who have had access to the manuscripts (Commandino, Wallis, Fortia d'Urban, Heath) are often reported in the first Latin translation of the Aristarchus's treatise by Giorgio Valla. For instance, in the proposition $11^{\circ}$ in the place of $1 / 30$ is written 30 ; in the proposition $12^{\circ}$ rather than 89 is written 890 ; in the proposition $13^{\circ}$ rather than 225 is written 25 , in the place of 979 is written 379 and in the place of 10125 is written 41125 . Different numbers are reported also in the propositions 16 and 18. Since Valla not touch upon the sources from which he has taken his translation, we can not know if the variances are due to printing error, to corrupted sources or, unlikely thing, to translation errors. Finally Valla's work is not very reliable. Perhaps for this reason Commandino does not mention it in his work.

Georgio Valla Placentino interprete: Hoc in volumine haec continetur Nicephori logica etc... Aristarchi Samii de Magnitudinibus et Distantiis Solis \& Lunae etc... Venetiis: Per Simone[m] Papiensem dictum Beuilaquam., 1498. Die ultimo Septembris.

## WORKS OF FEDERICO COMMANDINO

1558 Archimedis Opera Non Nulla a Federico Commandino Vrbinate nuper in Latinum conuersa, et commentariis illustrata. Quorum nomina in sequenti pagina leguntur. Cum privilegio in annos X. Venetiis, apud Paulum Manutium, Aldi F. M D LVIII.
Rainutio Farnesio, Cardinali Amplissimo et Optimo.
1558 Ptolemaei Planisphaerium. Iordani Planisphaerium. Federici Commandini Vrbinatis In Ptolemaei Planisphaerium Commentarius. In quo uniuersa Scenographices ratio quam breuissime traditur, ac demonstrationibus confirmatur. Venetiis: Aldus. M D LVIII.
Ranutio Farnesio Cardinali Amplissimo et Optimo
Claudii Ptolemaei Sphaerae A' planetis Proiectio in Plano.
1562 Claudii Ptolomaei Liber De Analemmate, a Federico Commandino Vrbinate instauratus \& commentariis illustratus, qui nunc primum eius opera e tenebris in lucem prodit. Eiusdem Federici Commandini liber de horologiorum descriptione. Romae A.D. M.D.LXII apud Paulum Manutium Aldi F.
Ranutio Farnesio Cardinali Amplissimo et Optimo.
1565 Archimedis De Iis Quae Vehuntur In Aqua Libri Duo. A Federico Commandino Vrbinate in Pristinum Nitorem Restituti, Et Commentariis Illustrati. Cum Privilegio in Annos X. Bononiae ex Officina Alexandri Benacii M D LXV.
Ranutio Farnesio Cardinali Amplissimo et Optimo
1565 Federici Commandini Urbinatis Liber De Centro Gravitatis Solidorum. Cum Privilegio in annos X. Bononiae ex Officina Alexandri Benacii M D LXV.
Alexandro Farnesio Cardinali Amplissimo et Optimo.
1566 Apollonii Pergaei Conicorum Libri Quattuor. Una cum Pappi Alexandrini Lemmatibus, et Commentariis Eutocii Ascalonitae. Sereni Antinsensis Philosophi Libri Duo nunc primum in lucem editi. Quae omnia nuper Federicus Commandinus Vrbinas mendis quamplurimis expurgata è Graeco conuertit, \& commentariis illustrauit. Bononiae ex officina Alexandri Benatii M D LXVI.

Vol 2. Sereni Antinsensis Philosophi Libri Duo. Vnus de sectione cylindri, alter de sectione coni. A Federico Commandino vrbinate e Graeco conuersi, et commentariis illustrati. Bononiae: ex officina Alexandri Benatii, M D LXVI.

1570 De Superficierum Divisionibus Liber Machometo Bagdedino Ascriptus nunc primum Joannis Dee Londinensis et Federici Commandini Vrbinatis opera in lucem editus. Federici Commandini de eadem re libellus Pisauri Apud Hyeronimus Concordiam Licentia Superiorum M D LXX. Illustrissimo atque Excellentissimo Francisco Maria II Vrbinati Principi. Federico Commandino Vrbinati Ioannes Dee Londinensis S.P.D.

1570 Libro Del Modo Di Dividere Le Superficie Attribuito A’ Machometo Bagdedino. Mandato in luce la prima volta da M. Giovanni Dee da Londra, e da M. Federico Commandino da Vrbino. Con vn breve trattato intorno alla stessa materia del medesimo M. Federico. Tradotti di latino in volgare da Fvlvio Viani de' Malatesti da Montefiore Academico Vrbinate e novamente dati in luce. In Pesaro del M D LXX.
Presso Girolamo Concordia con licentia de' Superiori
All'Illustrissimo et Eccellentissimo Signore il Sig. Francesco Maria II Principe D' Vrbino.

1572 Euclidis Elementorum Libri XV. Vna cum scholijs antiquis. A' Federico Commandino Vrbinate nuper in latinum conuersi, commentarijsque quibusdam illustrati. Pisauri M D LXXII Cum Privilegio Pont.Max.
Iacobus Chriegher German. (Finis: Pisauri cum licentia Superiorum: apud Camillum Francischinum, MDLXXII).
Illustrissimo atque Excellentissimo Francisco Mariae II Vrbinatum Principi.

1572 Aristarchi De Magnitudinibus et Distantiis Solis et Lunae Liber.
1575 Heronis Alexandrini Spiritalium Liber à Federico Commandino Vrbinate, ex Graeco in Latinum conuersus. Cum privilegio Gregorij XIII. Pont. Max. Urbini M D LXXV.
Illustrissimo ac Reuerendissimo Iulio Ruuerio Sanctae Ro.Eccl. Cardinali Amplissimo. Valerius Spaciolus. S.D.

1575 De Gli Elementi D'Euclide Libri Quindici. Con gli scholii antichi. Tradotti prima in lingua latina da M. Federico Commandino da Vrbino, \& con commentarij illustrati, et hora d'ordine dell'istesso trasportati nella nostra vulgare, \& da lui riueduti. Con privilegio. In Vrbino: appresso Domenico Frisolino. MCLXXV con licentia de' Superiori.
Finis: In Urbino in casa Di Federico Commandino con Licentia de’ Superiori M D LXXV.

1589 Pappi Alexandrini Mathematicae Collectiones a Federico Commandino Vrbinate in latinum conuersae, et commentariis illustratae Venetiis apud Franciscum De Franciscis Senensem M D LXXXIX Finis Pisauri: apud Hieronymum Concordiam, M D LXXXVIII.
Serenissimo Franc.co Mariae II Vrbini Duci Valerius Spacciolus s.p.d.

## INDEX OF NAMES

Paolus III. 15,17
Pappus of Alessandria. 7,8,13
Philolaus. 12
Pytagoras. 25
Ratdolt, Erhardus. 5
Reticus see Lauchen Georg Johachim von.
Scopas of Tarentum. 12
Serenus Antinsensis
(Sereno di Antinoeia or of Antissa or of Antinoupolis). 7
Spacciuoli, Valerio. 8
Tartaglia, Niccolò see
Fontana Niccolò.
Theodosius of Bithynia. 41
Ptolemy (Ptolemaeus). 6,7,16
Venatorius, Thomas see Gechauff Thomas.
Viani, Fulvio. 8
Vitruvius. 12
Walder, Johann. 6

## SUMMARY

## 5 INTRODUCTION

21 ARISTARCHUS'S BOOK ON THE SIZES AND

95 NOTES $1^{\circ}$ section

191 NOTES $2^{\circ}$ section
192 WORKS OF FEDERICO COMMANDINO
195 INDEX OF NAMES
$\square$

